

42nd International Mathematical Olympiad
Washington, D.C., United States of America
Day I 9 a.m. - 1:30 p.m.
July 8, 2001

1. Let ABC be an acute-angled triangle with O as its circumcenter. Let P on line BC be the foot of the altitude from A . Assume that $\angle BCA \geq \angle ABC + 30^\circ$. Prove that $\angle CAB + \angle COP < 90^\circ$.

2. Prove that

$$\frac{a}{\sqrt{a^2 + 8bc}} + \frac{b}{\sqrt{b^2 + 8ca}} + \frac{c}{\sqrt{c^2 + 8ab}} \geq 1$$

for all positive real numbers a, b , and c .

3. Twenty-one girls and twenty-one boys took part in a mathematical competition. It turned out that
- (a) each contestant solved at most six problems, and
 - (b) for each pair of a girl and a boy, there was at least one problem that was solved by both the girl and the boy.

Prove that there is a problem that was solved by at least three girls and at least three boys.

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4. Let n be an odd integer greater than 1 and let c_1, c_2, \dots, c_n be integers. For each permutation $a = (a_1, a_2, \dots, a_n)$ of $\{1, 2, \dots, n\}$, define $S(a) = \sum_{i=1}^n c_i a_i$. Prove that there exist permutations b and c , $b \neq c$, such that $n!$ divides $S(b) - S(c)$.
5. In a triangle ABC , let segment AP bisect $\angle BAC$, with P on side BC , and let segment BQ bisect $\angle ABC$, with Q on side CA . It is known that $\angle BAC = 60^\circ$ and that $AB + BP = AQ + QB$. What are the possible angles of triangle ABC ?
6. Let $a > b > c > d$ be positive integers and suppose

$$ac + bd = (b + d + a - c)(b + d - a + c).$$

Prove that $ab + cd$ is not prime.