Geometry

1. Let $\Delta ABC \sim \Delta A'B'C'$. Prove that the following are equivalent:
   (a) $I = I'$
   (b) $a\overrightarrow{AA'} + b\overrightarrow{BB'} + c\overrightarrow{CC'} = \overrightarrow{0}$.

2. Let ΔABC and $\Delta A'B'C'$. Prove that: $3\overrightarrow{GG'} = 2\overrightarrow{OO'} + \overrightarrow{HH'}$.

3. Let $\Delta ABC \sim \Delta A'B'C'$ and $I = I'$, $G = G'$. If any two of the vectors $\overrightarrow{AA'}$, $\overrightarrow{BB'}$, $\overrightarrow{CC'}$ aren't collinear prove that ΔABC and $\Delta A'B'C'$ are equilateral.

4. Let ΔABC and A', B', C' the points in which the angle bisectors intersect the opposite sides. Prove that if $\Delta ABC \sim \Delta A'B'C'$ and $I = I'$ then ΔABC and $\Delta A'B'C'$ are equilateral.

5. Let $ABCD$ be a quadrilateral, $\{O\} = AC \cap BD$ and M a point all in the plane $\mathbf{P}$. Consider the relation: $4\overrightarrow{MM'} = 2\overrightarrow{MO} + \overrightarrow{MA} + \overrightarrow{MB} + \overrightarrow{MC} + \overrightarrow{MD}$.
   (a) Prove that there exists the bijective function $f : \mathbf{P} \to \mathbf{P}$ so that $f(M) = M'$ for every $M \in \mathbf{P}$.
   (b) Prove that all lines MM' pass trough a fixed point and determine it.

6. Prove that if the altitudes of a triangle can form another triangle than the initial triangle is equilateral.

7. Let ΔABC and M the midpoint of AB. Let M' be its projection on the side AC and denote the centroid of $\Delta MM'A$ by G_1. Points G_2, G_3 are obtained analogously. Prove that if ΔABC and $\Delta G_1G_2G_3$ have the same centroid then they are equilateral.

8. Let ΔABC and A' the projection of A on BC. Let now M, N be the projections of A' on AB, AC respectively. Denote the centroid of ΔAMN by G_1. Points G_2, G_3 are obtained analogously. Prove that if ΔABC and $\Delta G_1G_2G_3$ have the same centroid then they are equilateral.

All problems are proposed by **Călin Popa**.