Functions

1. Determine the strictly increasing functions f:$\mathbf{R}_+ \to \mathbf{R}_+$ so that for every x,$y \in \mathbf{R}_+$ we have $f(x^2 + f^2(y)) = f(x^2) + y^2$.

2. Determine the functions f:$\mathbf{Z} \to \mathbf{Z}$ so that for every $x, y \in \mathbf{Z}$ we have: $f(x) + f(y) = f(x + y) - 2xy + 1$ and $f(1) = 0$.

3. Determine the functions $f : \mathbf{Z} \to \mathbf{Z}$ if $f(x + 2) = \frac{2002}{f(x)}$ for every $x \in \mathbf{Z}$ and $f(8) = f(9) = 2$.

4. Determine the functions $f : \mathbf{R} \to \mathbf{R}$ so that $f(f(x) + y) = yf(x)$ for every $x, y \in \mathbf{R}$.

5. Determine the functions $f : \mathbf{R} \to \mathbf{R}$ so that $f(x^2) - f^2(y) = (x + y)(f(x) - y)$ for every $x, y \in \mathbf{R}$.

6. Determine the functions $f : \mathbf{Z} \to \mathbf{Z}$ if $f(f(x+1)+y) = f(x+y)+1$ for every $x, y \in \mathbf{Z}$.

7. Determine the functions $f : \mathbf{R} \to \mathbf{R}$ so that $f(xy + af(z)) = zf(a) + y$ for every $x, y, z \in \mathbf{R}$ and $a \in \mathbf{R}$ is a constant. Which are the possible values of a?

8. Determine the functions $f : \mathbf{R} \to \mathbf{R}$ so that $xf(x) + y \geq f(f^2(x) + y) \geq x^2 + y$ for every $x, y \in \mathbf{R}$.

9. Determine the functions $f : \mathbf{N} \to \mathbf{N}$ so that $f(f(n) + 1) = n - 3$ for every $n \in \mathbf{N}$.

All problems are proposed by **Călin Popa**.