

The 9<sup>th</sup> Balkan Mathematical Olympiad  
Athens, Greece, 1992

1. Let  $m, n$  be positive integers and  $A(m, n) = m^{3^{4m}+6} - m^{3^{4m}+4} - m^5 + m^3$ . Find all  $n$  so that for every  $m$  we have  $A(m, n)$  is divisible by 1992.
2. Prove that for every positive integer  $n$  we have  $(2n^2 + 3n + 1)^n \geq 6^n (n!)^2$ .
3. Let  $ABC$  be a triangle and  $D, E, F$  be points on sides  $BC, CA, AB$ , different from the vertices, so that the quadrilateral  $AFDE$  is cyclic. Prove that 
$$4 \frac{\text{area}(DEF)}{\text{area}(ABC)} \leq \left( \frac{EF}{AD} \right)^2.$$
4. For every integer  $n \geq 3$  find the smallest positive integer  $f(n)$  that has the property: for every  $f(n)$  numbers in the set  $\{1, 2, \dots, n\}$ , one can find among these three numbers  $x, y, z$  that are pair-wisely prime.