The 8th Balkan Mathematical Olympiad
Constanta, Romania, 1991

1. Let M be a point on the small arc AB of the circumcircle of an acute-angled triangle ABC. The perpendicular from M to the ray OA meets the side AB and AC at K and L and the perpendicular from M to ray OB meets the sides AB and AC at N and P. If KL = MN then find MLP as a function of the angles of ABC.

2. Prove that here exists an infinite set of triangles T which are pair-wisely non-congruent and which fulfill the conditions:
   - The lengths of the sides of T are positive integers, pair-wisely relatively prime.
   - The area of T is an integer
   - None of the altitudes of T is an integer.

3. A regular hexagon having area H has its vertices on a regular n – gon of area P. Prove that $P \leq 3H/2$. When does equality hold?

4. Prove that there is no bijection f from the set of positive integers to the set of non-negative integers so that for all $m, n \geq 1$ we have $f(m \cdot n) = f(m) + f(n) + 3f(m)f(n)$.