The 7th Balkan Mathematical Olympiad
Sophia, Bulgaria, 1990

1. Let $a_1 = 1, a_2 = 3, a_{n+2} = (n+3)a_{n+1} - (n+2)a_n$. Find n so that a_n is divisible by 11.

2. A polynomial is given by the relationship: $a_0 + a_1 x + \ldots + a_{2n} x^{2n} = (x + 2x^2 + \ldots + nx^n)^2$. Prove that $a_{n+1} + \ldots + a_{2n} = \dfrac{n(n+1)(5n^2 + 5n + 2)}{24}$.

3. Let MNP be the orthic triangle of an acute-angled non-equilateral triangle ABC, and D, E, F the points of tangency of the incircle of MNP with sides MN, NP, PM. Prove that the triangles ABC, DEF have the same Euler line.

4. Find the minimum number of elements of a finite set A so that one may define a function from the set of non-negative integers to A with the property: if $p - q$ is a prime then $f(p)$ and $f(q)$ are different.