The 6th Balkan Mathematical Olympiad
Split, 1989

1. Let d_1, …, d_k be all the natural divisors of a positive integer n. We have $1 = d_1 < d_2 < \ldots < d_k = n$. Find n so that $n = d_1^2 + d_2^2 + d_3^2 + d_4^2$.

2. Let $a_n a_{n-1} \ldots a_1 a_0$ be the decimal expansion of a prime. If $n > 1$ and $a_n > 1$ then the polynomial $P = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$ is irreducible over Z[x].

3. Let ABC be a triangle of centroid G and l a line that intersects AB and AC at B_1 and C_1, so that A and G are on the same side of l. Prove that $area(BB_1GC_1) + area(CC_1GB_1) \geq 4/9 area(ABC)$ and find the cases of equality.

4. Let F be a family of subsets of $\{1, 2, \ldots, n\}$. If A ∈ F then #A = 3. If A, B ∈ F then A and B have at most 1 common element. Let $f(n)$ be the greatest number of elements of F. Prove that $n(n-3)/2 \leq f(n) \leq n(n-1)/2$.