The 5th Balkan Mathematical Olympiad
Nicosia, Cyprus, 1988

1. Let CH, CK and CM be the altitude, bisector and median from C in triangle ABC. If $area(HMC)/area(ABC) = 1/4$ and $area(LMC)/area(ABC) = 1 - \frac{\sqrt{3}}{2}$. Find the angles of ABC.

2. Find all two-variable polynomials P so that for all a, b, c, d we have $P(a, b)P(c, d) = P(ac + bd, ad + bc)$

3. Prove that any tetrahedron ABCD can be included in the region between two planes so that the distance between the two planes is $\leq \frac{\sqrt{\sum AB^2}}{2\sqrt{3}}$.

4. Find all consecutive members of the sequence (a_n) defined by $a_n = 2^n + 49$ so that $a_n = pq$ and $a_{n+1} = rs$, where p, q, r, s are prime numbers so that $p < q$, $r < s$, $q - p = s - r$.