

The 4th Balkan Mathematical Olympiad
Athens, Greece, 1987

1. Let a be a real number and f a real function so that $f(0) = 1/2$ and for any real x, y we have $f(x + y) = f(x)f(a - y) + f(a - x)f(y)$. Prove that f is constant.
2. Find all real x, y so that $\sqrt{x-1} + \sqrt{y-1}, \sqrt{x+1} + \sqrt{y+1}$ are consecutive integers.
3. Find AC/BC if in triangle ABC we have $\sin^{23} A/2 \cos^{48} B/2 = \sin^{23} B/2 \cos^{48} A/2$.
4. Two circles K and L , of centers O and Q and radii 1 and $\sqrt{2}$, respectively, intersect at A and B . $OQ = 2$ and C is a point on L so that the midpoint of AC is on K . Find the length of AC .