The 2nd Balkan Mathematical Olympiad
Sophia, Bulgaria, 1985

1. Let O be the circumcenter of ABC, D the midpoint of AB and E the centroid of ACD. Prove that CD and OE are perpendicular if and only if AB = AC.

2. Let a, b, c, d be real numbers in the interval $[-\pi/2, \pi/2]$ so that $\sin a + \sin b + \sin c + \sin d = 1$, and $\cos 2a + \cos 2b + \cos 2c + \cos 2d \geq 10/3$. Prove that $a, b, c, d \in [0, \pi/6]$.

3. On the axis E take the set S of points of coordinates $19a + 85b$ where a, b are positive integers. The points of S are colored red, whereas all the other points of E are colored black. Find whether there is a point M on E so that any two lattice points of E, symmetric with respect to M, are colored differently.

4. 1985 people participate at a reunion. In any group of three at least two people speak a common language. Knowing that each person at the reunion speaks at most five languages, prove that there are at least 200 people speaking a same language.