

19th BALKAN MATHEMATICAL OLYMPIAD
Turkey, 2002

1. Let G be an unoriented graph so that $d(x) \geq 3$ for any vertex x of G . Prove that G contains an even cycle.
2. Consider the sequence $(a_n)_{n \geq 1}$ as follows so that $a_1 = 20, a_2 = 30$ and $a_{n+1} = 3a_n - a_{n-1}$ for $n \geq 2$. Find all n so that $1 + 5a_n a_{n+1}$ is a perfect square.
3. Consider two non-equal circles which intersect in A and B . Let their common tangents touch the first circle in M, N and the second one in S, T . Prove that the orthocentres of triangles $\triangle AMN, \triangle BMN, \triangle AST, \triangle BST$ are the vertices of a rectangle.
4. Find all functions $f : \mathbb{N} \rightarrow \mathbb{N}$ which satisfy the inequality:

$$2n + 2001 \leq f(f(n)) + f(n) \leq 2n + 2002$$