

The 18th Balkan Mathematical Olympiad
Belgrade, Serbia, 2001

1. If $ab = 2^n - 1$ then $ab - (a - b) - 1$ can be written as $2^{2m}(2u + 1)$.
2. ABCDE is a convex pentagon with equal angles and rational side lengths. Prove that ABCDE is a regular pentagon.
3. Let a , b and c be three positive numbers so that $a + b + c \geq abc$. Prove that $a^2 + b^2 + c^2 \geq abc\sqrt{3}$.
4. A $3 \times 3 \times 3$ cube is divided into 27 unit cubes. One of these cubes is removed. The other cubes are assigned arbitrarily the numbers 1, 2, ..., 26. At every moment, the initial cube has 26 unit cubes and one missing one. A valid move is a move that takes a unit cube adjacent to the unit cube missing into its place. This way an empty cube appears in the place of the cube that moved. Can a finite sequence of valid moves switch the position of cube i with the one of cube $27 - i$, for all $i = 1, 2, \dots, 13$?