15th BALKAN MATHEMATICAL OLYMPIAD
Nicosya, Cyprus, 1998

1. Find the number of different terms of the finite sequence $\left[\frac{k^2}{1998}\right]$, where k = 1, 2, ..., 1997 and $[x]$ denotes the integer part of x.

2. If $n \geq 2$ is an integer and $0 < a_1 < a_2 < \ldots < a_{2n+1}$ are real numbers, prove the inequality:
$$\sum_{k=1}^{2n+1}(-1)^k \sqrt[n]{a_k} < \sqrt[n]{\sum_{k=1}^{2n+1}(-1)^k a_k}$$

3. Let S be the set of all points inside and on the border of triangle ABC without one inside point T. Prove that S can be represented as a union of closed segments no two of which have a point in common. (A closed segment contains both of its ends).

4. Prove that the equation $y^2 = x^5 - 4$ has no integer solutions.