# $14^{th}$ BALKAN MATHEMATICAL OLYMPIAD

Kalampaka, Greece, April 29, 1997

1. Let $O$ be an interior point of a convex quadrilateral $ABCD$ such satisfying $OA^2 + OB^2 + OC^2 + OD^2 = 2area(ABCD)$. Show that $ABCD$ is a square with center $O$.

2. Let $A = \{A_1, A_2, ..., A_k\}$, (k>1), be a collection of subsets of an *n-set* $S$ such that for any $x, y \in$ S there is $A_i \in A,$ such that $x \in A_i$ and $y \in A_i$ or $x \in A_i$ and $y \in A_i$. Show that $K \geq [\log_2 n]$.

3. Three circles $\Gamma$, $C_1$ and $C_2$ are given in the plane. $C_1$ and $C_2$ touch $\Gamma$ internally at points $B$ and $C$, respectively. Moreover $C_1$ and $C_2$ touch each other externally at a point $D$. Let $A$ be one points in which the common tangent of $C_1$ and $C_2$ intersects $\Gamma$. Denote by M the second point of intersection of the line $AB$ and the circle $C_1$, and by $N$ the second point of intersection of the line $AC$ and the circle $C_2$. Furthermore, denote by $K$ and $L$ second points of intersections of the line $BC$ with $C_1$ and $C_2$, respectively. Show that lines $AD$, $MK$ and $NL$ are concurrent.

4. Find all functions $f$ : R → R such that $f(xf(x) + f(y)) = (f(x))^2 + y$ for all real $x$ and $y$.