13th BALKAN MATHEMATICAL OLYMPIAD
Bacau, Romania, 1996

1. Let ABC be a triangle with circumcenter O, centroid G, inradius r and circumradius R. Prove that $OG \leq \sqrt{R(R-2r)}$.

2. Let p a prime number greater than 5. Prove that the set X = $\{p - n^2 \,/\, n$ integer and $n^2 \geq p\}$ contains two different elements x and y, different from 1, such that x divides y.

3. Let ABCDE be a convex pentagon and M, N, P, Q, S be the midpoints of its sides AB, BC, CD, DE, EA. If the lines DM, EN, AP and BQ have a common point then this point belongs to CS as well.

4. Find whether there is a subset A of the set $\{1, 2, 3, \ldots, 2^{1996} - 1\}$ with at most 2012 elements such that 1 and $2^{1996} - 1$ both belong to A, and every element of $A \setminus \{1\}$ is the sum of two not necessary distinct elements of A.