12^{th} BALKAN MATHEMATICAL OLYMPIAD

Plovdiv, Bulgaria, 1995

1. Find the value of the expression $(\ldots(((2\circ 3)\circ 4\circ 5)\circ\ldots)\circ 1995$, where $x\circ y=\frac{x+y}{1+xy}$.

2. Consider two circles C_1 and C_2 with centers O_1, O_2 and radii r_1, r_2, respectively ($r_2>r_1$) which intersect at A and B such that $\angle O_1AO_2=90°$. Line O_1O_2 intersects C_1 at C and D, and C_2 at E and F, where E lies between C and D and D lies between E and F. Line BE meets C_1 at K and intersects line AC at M, and BD meets C_2 at L and intersects line AF at N. Show that $\frac{KE}{KM}\frac{LN}{LD}=\frac{r_2}{r_1}$.

3. Let a,b be positive integers such that $a>b$ and $a+b$ is even. Prove that the roots of the equation $x^2-(a^2-a+1)(x-b^2-1)-(b^2+1)^2=0$ are positive integers, none of which is a perfect square.

4. Let n be a positive integer and S the set of all points (x,y), where x and y are positive integers with $x,y\leq n$. Assume that T is the set of all squares whose vertices belong to S. Denote by a_k $(k\geq 0)$ the number of pairs of points in S which are the vertices of exactly k squares from T. Prove that $a_0=a_2+2a_3$.