

11<sup>th</sup> BALKAN MATHEMATICAL OLYMPIAD  
Novi Sad, Yugoslavia, 1994

1. Consider the angle XOY and P inside this angle. Construct the line  $d$  passing through P using the ruler and the compass such that the area of OAB is  $OP^2$  where A and B are the intersections of OX and OY with  $d$ .
2. Prove that the polynomial  $x^4 - 1993x^3 + (1993 + m)x^2 - 11x + m$  has at most one integer root.
3. If  $(a_1, a_2, \dots, a_n)$  is a permutation of the set  $\{1, 2, \dots, n\}$  where  $n$  is a fixed positive integer, find the maximum value of the sum  $|a_1 - a_2| + |a_2 - a_3| + \dots + |a_{n-1} - a_n|$
4. Find the least integer  $n > 4$  such that there exists a set of  $n$  persons with the following properties:
  - for any two persons that do not know each other there are no common acquaintances
  - any two persons that do not know each other have exactly two common friends.It is assumed that if A has B as his friend then also B has A as its friend.