The 10th Balkan Mathematical Olympiad
Athens, Greece, 1992

1. Given a, b, c, d, e, f real numbers so that $a + b + c + d + e + f = 10$ and $(a-1)^2 + (b-1)^2 + (c-1)^2 + (d-1)^2 + (e-1)^2 + (f-1)^2 = 6$, find the greatest value of f.

2. A positive integer with decimal representation $a_n a_{n-1} \ldots a_1 a_0$ is called monotonous if $a_n \le a_{n-1} \le \ldots \le a_1 \le a_0$. Find the number of monotonous numbers with at most 1993 digits.

3. Let C_1, C_2 be two externally tangent circles with centers O_1, O_2, the later lying outside C_1. They touch at T. Consider a circle C tangent to C_1 and C_2 at A and B, respectively, so that the centers O_1, O_2 are inside C. The center of C is O. The common tangent to C_1 and C_2 at T meets C at K and L. If D is the midpoint of KL show that angles O_1OO_2 and ADB are congruent.

4. Let p be a prime number and m a positive integer ≥ 2. Show that the equation $\frac{x^p + y^p}{2} = \left(\frac{x+y}{2}\right)^m$ has a solution (x, y) different from $(1, 1)$ if and only if $m = p$.