The 1st Balkan Mathematical Olympiad
Athens, Greece, 1984

1. Let $a_1, \ldots, a_n$ be positive real numbers with sum 1. Prove that $\sum_{i=1}^{n} \frac{a_i}{1+\sum_{j\neq i} a_j} \geq \frac{n}{2n-1}$.

2. Let $A_1A_2A_3A_4$ be a cyclic quadrilateral. Let H_i be the orthocenter of $A_kA_lA_m$, where (i, k, l, m) is a permutation of (1, 2, 3, 4). Prove that the quadrilaterals $A_1A_2A_3A_4$ and $H_1H_2H_3H_4$ are congruent.

3. Prove that for any positive integer m there is an $n > m$ so that the decimal expansion of 5^n can be obtained by placing some digits to the left of the decimal expansion of 5^m.

4. Find the solution to the system: $ax + by = (x - y)^2$, $by + cz = (y - z)^2$, $cz + ax = (z - x)^2$, where a, b, c are positive real numbers.