6^{th}JUNIOR BALKAN MATHEMATICAL OLYMPIAD

1. Let ΔABC be an isosceles triangle so that $BC = AC$ and P a point on the arc $\widehat{AB}$ of the circumscribed circle of ΔABC, which doesn't contain point C. Let D be the projection of C on PB. Prove that $PA + PB = 2 \cdot PD$.

2. The non-congruent circles $\mathbf{C}_1, \mathbf{C}_2$ intersect in points A, B so that AB separates the centers $\mathbf{O}_1, \mathbf{O}_2$. Let B_1, B_2 be the ends of the diameters from B in the circles $\mathbf{C}_1, \mathbf{C}_2$ and M the midpoint of B_1B_2. On $\mathbf{C}_1, \mathbf{C}_2$ consider the points M_1, M_2 so that $\angle AO_1M_1 \equiv \angle AO_2M_2$, where B_1 is interior to $\angle AO_1M_1$ and B_2 is interior to $\angle AO_2M_2$. Prove that $\angle MM_1B \equiv \angle MM_2B$.

3. Determine the positive integer n so that:

   (a) n has exactly 16 divisors: $1 = d_1 < d_2 < ... < d_{16} = n$;

   (b) $d_{d_5} = (d_2 + d_4) \cdot d_6$.

4. Prove that:

$$\frac{1}{b(b+a)} + \frac{1}{c(c+b)} + \frac{1}{a(a+c)} \geq \frac{27}{2(a+b+c)^2}, \forall a, b, c \in (0, \infty).$$