5^{th} JUNIOR BALKAN MATHEMATICAL OLYMPIAD

1. Solve in positive integers the equation: $a^3 + b^3 + c^3 = 2001$.

2. Let ΔABC with $m(\angle ACB) = 90^\circ$ and $AC \neq BC$. Points $L, K \in [AB]$ are chosen so that $m(\angle ACL) = m(\angle LCB)$ and $CH \perp AB$.

   (a) For every point $X \neq C, X \in CL$ prove that $m(\angle XAC) \neq m(\angle XBC)$.

   (b) For every point $Y \neq C, Y \in CH$ prove that $m(\angle YAC) \neq m(\angle YBC)$.

3. Let ΔABC an equilateral triangle and D, E arbitrary points on the sides $[AB], [AC]$ respectively. If $DF, EG (F \in AE, G \in AD)$ are the interior bisectors of the angles of ΔADE, prove that $[DEF] + [DEG] \leq [ABC]$. When does the equality hold? ([XYZ] denotes the area of the triangle XYZ)

4. A convex polygon with 1415 sides has the perimeter 2001 cm. Prove that there are three vertices of the polygon so that the triangle that they form has the area less than 1 cm^2.

Compiled by **Călin Popa**.