1^{st} American Selection Test for the 43^{rd} IMO

1. Prove that if A, B, C are a triangle's angles, then the following inequality holds:

$$\cos\frac{A-B}{2}+\cos\frac{B-C}{2}+\cos\frac{C-A}{2}\geq \sin\frac{3A}{2}+\sin\frac{3B}{2}+\sin\frac{3C}{2}.$$

2. Let p be a prime greater than 5. Let $f:\mathbf{R}\to\mathbf{R}$ be a function such that:

$$f(x)=\sum_{k=1}^{p-1}\frac{1}{(px+k)^2}.$$

   Prove that if x, y are integers then $f(x)-f(y)$ has the numerator divisible with p^3 , when it's written in lowest terms.

3. Let $n>2$ be a positive integer and $P_1, ..., P_n$ distinct points in the plane. Let $S=\bigcup_{i=1}^{n} P_iP_{i+1}$. Determine if one can always find $A, B\in S$ so that $P_1P_n || AB$ (AB can be on P_1P_n) so that $P_1P_n = kAB$ when:

   (a) $k=2.5$

   (b) $k=3$.

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