3^{rd} JUNIOR BALKAN MATHEMATICAL OLYMPIAD

1. Let $a, b, c, x, y \in \mathbf{R}$ so that $a^3 + ax + y = 0$, $b^3 + bx + y = 0$, $c^3 + cx + y = 0$. If $a \neq b \neq c \neq a$, prove that $a + b + c = 0$.

2. For $n = 0, 1, ..., 1999$ define $A_n = 2^{3n} + 3^{6n+2} + 5^{6n+2}$. Find the greatest common divisor of the numbers $A_0, A_1, ..., A_{1999}$.

3. Let S be a square with the side of length 20. Let M be the set of the vertices of S and other 1999 points from inside S. Prove that there exists a triangle with vertices in M and with the area at most equal to $\frac{1}{10}$.

4. Let ΔABC with $[AB] \equiv [AC]$. Let $D \in [BC]$ be an arbitrary point so that $BC > BD > DC > 0$. Let k_1, k_2 be the circumscribed circles of $\Delta ABD, \Delta ADC$. Let $[BB'], [CC']$ be the diameters of k_1, k_2 and M the midpoint of $(B'C')$. Prove that the area of ΔMBC is a constant, it doesn't depend on the position of M.

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