ROMANIAN MATHEMATICAL OLYMPIAD
FINAL ROUND - 1988

$\mathbf{X^{th}}$ FORM

1. Prove that if a, b are two complex numbers so that $ta + (1-t)b \neq 0, \forall t \in [0,1]$ and $\Re \frac{a}{b} \geq 0$ then $\Re \frac{ab}{[ta+(1-t)b]^2} > 0, \forall t \in (0,1)$.

2. Prove that a nonconstant sequence of positive integers $(x_n)_{n\geq 0}$ is an arithmetical progression iff there exists $a > 0$ so that $x_{n+1} = x_n + \left[\frac{x_n}{n+a}\right], \forall n \in \mathbf{N}$.

3. For $n \in \mathbf{N}^*$ consider the numbers $\left(\begin{array}{c} 2^n \\ k \end{array}\right), k = 1, 2, ..., 2^n - 1$. Prove that:

   (a) all these numbers are even;

   (b) a single number isn't divisible by 4.

4. Consider a regular pyramid of base $ABCD$ and vertex V. Consider the points $A' \in (VA), B' \in (VB), C' \in (VC), D' \in (VD)$ and denote the midpoints of $[A'C'], [B'D']$ by M, N. Prove that the points V, A', B', C', D' are on a sphere iff $MN || (ABC)$.