ROMANIAN MATHEMATICAL OLYMPIAD
FINAL ROUND - 1985

X$^{\text{th}}$ FORM

1. (a) Determine two integer solutions of the equation: $3^{2^{x!}} = 2^{3^{x!}} + 1$.

   (b) Solve the equation in positive integers.

2. Let $z_1, z_2, z_3 \in \mathbf{C}$ so that $z_1 + z_2 + z_3 \neq 0$, $z_1^2 + z_2^2 + z_3^2 = 0$ and $|z_1| + |z_2| + |z_3| = 1$. Prove that $|z_1 + z_2 + z_3| = 2$.

3. If a, b, c are the lengths of the sides of a triangle prove that: $a(2a^2 - b^2 - c^2) + b(2b^2 - c^2 - a^2) + c(2c^2 - a^2 - b^2) \geq 0$ and determine when the equality holds.

4. Let ΔABC and π a plane which doesn't contain the triangle.

   (a) Which is the necessary and sufficient condition so that the projection of ΔABC on π is a triangle?

   (b) If the projection of ΔABC on π is a triangle $\Delta A'B'C'$ prove that the projection of the orthocenter of ΔABC on π is the orthocenter of $\Delta A'B'C'$ iff the projection of the circumcenter of ΔABC is the circumcenter of $\Delta A'B'C'$.