ROMANIAN MATHEMATICAL OLYMPIAD
FINAL ROUND - 1984

$\mathbf{X^{th}}$ FORM

1. Determine the strictly monotone functions $f : \mathbf{Z} \to \mathbf{Z}$ with the propriety that $f(xy) = f(x)f(y), \forall x, y \in \mathbf{Z}$ and there exists $k \in \mathbf{Z}, |k| \geq 2$ so that $f(k) = k$.

2. Determine the minimum value of the expression:

$$\log_{x_1}\left(x_2 - \frac{1}{4}\right) + \log_{x_2}\left(x_3 - \frac{1}{4}\right) + ... + \log_{x_n}\left(x_1 - \frac{1}{4}\right),$$

   where $x_1, x_2, ..., x_n \in \left(\frac{1}{4}, 1\right)$ and determine the values of $x_1, x_2, ..., x_n$ for which this minimum is obtained.

3. Let $k \geq 1, k \in \mathbf{N}$ and $a_0, a_1, ..., a_k \in \mathbf{Q}$ so that not all $a_0, a_1, ..., a_k$ are 0. Consider the sequence $(x_n)_{n\in\mathbf{N}}$ so that: $x_n = a_k n^k + a_{k-1}n^{k-1} + ... + a_1 n + a_0, n \in \mathbf{N}$. Prove that this if sequence contains an integer number then it contains infinitely many distinct integers.

4. In ΔABC with $m(\angle A) = 90°$ denote $AB = c, BC = a, CA = b$. Let M be a point in space so that $MA = x, MB = y, MC = z$. Prove that $M \in (ABC)$ iff:

$$2(y^2 + z^2) = a^2 + \frac{(x^2 - y^2)^2}{c^2} + \frac{(x^2 - z^2)^2}{b^2}.$$